

Exercise Sheet 3 for Algorithmen und Datenstrukturen (Sommer 2026)

Hand In: Until 2026-05-08 18:00, on ILIAS.

Problem 1

30 points

a) Formally prove the following:

$$\binom{n}{3} \sim \frac{n^3}{6} \quad (1)$$

$$\ln^2(n) = o(\sqrt{n}) \quad (2)$$

$$3^n = \omega(2^n) \quad (3)$$

b) Give the correct set-theoretic formulation (involving sets of functions) of the following statements

$$n^3 = \Omega(2 + \sin(n)) \quad (4)$$

$$\lg(n!) = n \lg n \pm O(n) \quad (5)$$

$$\Theta(f(n)) + \Theta(g(n)) = \Theta(\max\{f(n), g(n)\}) \quad (6)$$

Problem 2

40 points

Throughout let $-1 < \alpha < 1$. Give derivations for the following infinite series.

a) Compute $\sum_{k=0}^{\infty} \alpha^k$. Assume that this series converges.

b) Compute $\sum_{k=0}^{\infty} (k+1)\alpha^k$. Assume that this series converges. You may furthermore

assume that for our case $\sum_{k=0}^{\infty} \frac{\partial}{\partial x} f(k, x) = \frac{\partial}{\partial x} \sum_{k=0}^{\infty} f(k, x)$.

Hint: Reduce to from the previous case by differentiation with respect to α .

c) Similarly, compute $\sum_{k=0}^{\infty} (k+1) \cdots (k+t)\alpha^k$.

Problem 3

40 points

Give tight asymptotic upper bounds for the following recurrence relations. In each case, assume $T(n) = 1$ for $n \leq 1$.

a) $T(n) = 1 + T(n/2)$.

b) $T(n) = n + 2T(n/2)$.

c) $T(n) = \sqrt{n} + 2T(n/3)$.

d) $T(n) = \sqrt{n} + T(n/3)$.

e) For the following, give a non-trivial upper bound of form “for some c , for every ε , we have $T(n) = O(n^{c+\varepsilon})$ ”. $T(n) = T(n/2) + T(n/3) + T(n/4)$.