

Exercise Sheet 5 for Algorithmen und Datenstrukturen (Sommer 2026)

Hand In: Until 2026-05-23 18:00, on ILIAS.

Problem 1

30 points

Consider three algorithms A_1, A_2, A_3 , running on inputs of size n . Suppose that these running times of these algorithms, for various input sizes n . For each, decide if the running time is polynomial or exponential, and give a reasonable guess as to the running time.

n	A_1	A_2	A_3
1000	2.16	3.42	14.07
2000	10.12	10.80	20.09
3000	23.15	20.78	27.45
4000	39.45	31.68	40.87
5000	62.21	44.06	56.34
6000	89.55	59.20	79.93
7000	122.07	74.79	113.79
8000	160.33	91.47	159.65
9000	201.96	108.73	226.54
10000	249.62	127.12	319.91
11000	302.60	145.99	452.94
12000	359.69	167.10	639.38
13000	422.04	187.84	905.70
14000	489.27	208.60	1280.30
15000	562.19	232.19	1810.56
16000	639.55	256.01	2559.85
17000	721.89	279.81	3620.77
18000	810.27	304.60	5120.41
19000	901.62	331.36	7239.84

Problem 2

20 + 20 + 20 points

What is the correct asymptotic time complexity of the following algorithms, in terms of n ? Assume arithmetic operations take constant time.

$f(A[1..n])$:

```

1  if  $n \leq 100$ 
2      return 100
3  end if
4   $ret := 0$ 
5  for  $i := 1, \dots, n$ 
6      for  $j := i + 1, \dots, n$ 
7           $ret := ret + A[i] \cdot A[j]$ 
8      end for
9  end for
10  $ret := ret + f(A[1.. \lfloor n/2 \rfloor])$ 
11  $ret := ret + f(A[\lfloor n/6 \rfloor .. \lfloor 2n/3 \rfloor])$ 
12  $ret := ret + f(A[\lfloor n/3 \rfloor .. \lfloor 5n/6 \rfloor])$ 
13  $ret := ret + f(A[\lfloor n/2 \rfloor .. n])$ 
14 return  $ret$ 

```

$g(n, x)$:

```

1  if  $n \leq 100$ 
2      return 100
3  end if
4   $ret := 0$ 
5  for  $i := 1, \dots, \lfloor \sqrt{n} \rfloor$ 
6       $ret := ret + x \cdot i$ 
7  end for
8  return  $g(\frac{n}{4}, \frac{ret}{2}) + g(\frac{n}{4}, \frac{ret}{3}) + g(\frac{n}{4}, \frac{ret}{4})$ 

```

$h(A[1..n])$:

```

1  if  $n \leq 100$ 
2      return 100
3  end if
4   $ret := 0$ 
5  for  $i := 1, \dots, \lceil \sqrt{n} \rceil$ 
6      for  $j := i^2, \dots, (i + 1)^2 - 1$ 
7           $ret := ret + A[i] \cdot A[j]$ 
8      end for
9  end for
10  $ret := ret + h(A[1.. \lfloor n/3 \rfloor])$ 
11 return  $ret$ 

```