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Exercise Sheet 9 for Advanced Algorithms, Summer 2025

Hand In: *Until 2025-07-16 18:00, on ILIAS.*

Problem 1

20 points

We routinely use Markov's inequality to show that giving an algorithm twice as long as its expected running time has probability at least $\frac{1}{2}$ to successfully run to completion.

One specific use case of that pattern is an algorithm A which repeatedly calls another algorithm B that succeeds only with probability $q < \frac{1}{2}$. Denote by A_r the algorithm that uses exactly r independent calls (with fresh randomness) to B, and assume that A is successful if at least one of the r calls to B is successful. Then $r = \lceil 2/q \rceil$ is sufficient for overall success probability at least $\frac{1}{2}$.

Show that actually, $r = \lceil \ln(2)/q \rceil \le \lceil 0.7/q \rceil$ is also sufficient for overall success probability at least $\frac{1}{2}$.

Problem 2

30 points

Consider the following approach for solving Vertex Cover:

> Compute a spanning forest F of G by depth-first search and return the set of all inner nodes of F as result.

Show that this is a 2-approximation for Vertex Cover.

Problem 3

30 points

Consider the following problem P:

Input: Digraph $G = (V, E)$.

Solutions: Acyclic spanning (but not necessarily connected) subgraph $G' = (V, E')$ of G.

Goal: Maximise $|E'|$.

And furthermore the algorithm A:

1. Shuffle V, i.e., draw uniformly at a random a total order $\prec$ on V.
2. Flip a fair coin C.
3. Depending on C do:
   - If $C = 1$ use all *forward* edges (w. r. t. $\prec$), i.e., E' consists of all edges (u, v) with $u \prec v$.
   - If $C = 0$ use all *backward* edges, i.e., E' consists of all edges (u, v) with $v \prec u$.
4. return (V, E').

Show that A is a randomized $\frac{1}{2}$-expected approximation for P.

Problem 4

20 points

Show that there is no $\epsilon > 0$ so that `layeringSetCover` is an $(f - \epsilon)$-approximation for Set-Cover, i. e. that f is tight.

Hint: Give a set of instances that contains infinitely many counterexamples for every $\epsilon > 0$.