

# Exercise Sheet 1 for Advanced Data Structures (Summer 2026)

*Hand In: Until 2026-05-01 18:00, on ILIAS.*

## Problem 1

40 points

Suppose you have a playlist containing  $n$  songs, represented as a sequence  $s_1, \dots, s_n$ . Each song has a unique label which never changes; furthermore it also has a quality score, initially 0.

**Slice-move:** Given  $1 \leq i \leq j \leq n$  and  $k \notin (i, j]$ , remove  $s_i, \dots, s_{j-1}$  from the playlist and put it after  $s_k$ .

**Reverse:** Given  $1 \leq i \leq j \leq n$ , reverse  $s_1, \dots, s_{j-1}$ .

**Change-quality:** Given  $1 \leq i \leq j \leq n$  and  $\delta$ , increase the quality of songs  $s_i, \dots, s_{j-1}$  by  $\delta$ .

**Rank-query:** Given  $i$ , output the label of  $s_i$  as well as the current quality of  $s_i$ .

Give a data structure which implements all of these operations. Your data structure should furthermore work in (expected)  $O(\log n)$  time for every operation.

## Problem 2

30 points

Consider randomised skiplists with unbounded pointer tower height. Prove that the maximal height that occurs is at most  $O(\log n)$  with high probability. In particular: for every  $\alpha$ , the probability that this height exceeds  $\alpha \log_2(n)$  is at most  $1/n^{\alpha-1}$ .