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Exercise Sheet 4 for Advanced Data Structures (Summer 2026)

Hand In: *Until 2026-05-22 18:00, on ILIAS.*

Problem 1

40 points

Define the "bit reverse permutation" of order k as being the result of writing $0, \ldots, 2^k - 1$ down, but with their bits reversed. For example for $k = 3$, the bit reverse permutation is (in binary):

$$000, 100, 010, 110, 001, 101, 011, 111,$$

or in decimal

$$0, 4, 2, 6, 1, 5, 3, 7.$$

More formally, if we denote by $B(k)$ the bit reverse permutation of order k, we have $B(1) = \langle 0, 1 \rangle$, and $B(k+1) = (2 \times B(k)) \mid\mid (2 \times B(k) + 1)$, where $\times$ and $+$ operate pointwise and $\mid\mid$ denotes concatenation.

Compute a lower bound for the computational cost of doing accesses in a binary search tree with keys $0, \ldots, k - 1$, in bit reverse permutation order.

Problem 2

40 points

You have an orchard with n magic trees, arranged in a line. We denote by T_i the i-th tree in order. T_i has F_i fruits. You are initially beside T_1.

As time passes two types events may occur

- All the trees magically gain 1 extra fruit.
- You go from the tree you are currently at, say T_i, towards tree T_j, harvesting all the fruit along the way. Since you are not magic, you can only go so far: it is guaranteed that $|i - j| \leq \Delta$.

Give an algorithm which, given n, the original values of $F_1, \ldots, F_n$, and a list of m events as described above (i.e. whether an event is of the first kind or the second, and in the second case the target tree T_j to which you go), computes the total amount of fruit you harvest. For full marks your algorithm should run in $O(n + m \log \Delta)$.

Example: For example, we could have $n = 5$, $F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 4, F_5 = 5$ and $m = 3$. The first event says that we go from T_1 to T_3, harvesting $1 + 2 + 3 = 6$ fruits. The second event regrows one extra fruit on each tree. The third event says we go from T_3 to T_2, harvesting $1 + 1 = 2$ fruits. Overall we harvest $2 + 6 = 8$ fruits.