1 Machines & Models

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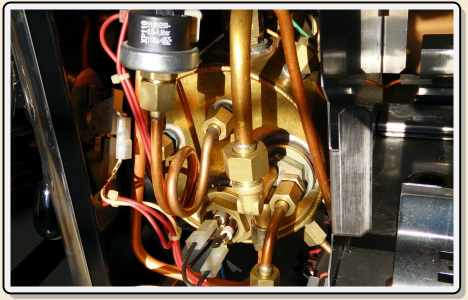
Learning Outcomes

1. Understand the difference between empirical *running time* and algorithm *analysis*.
2. Understand *worst / best / average case* models for input data.
3. Know the *RAM machine* model.
4. Know the definitions of *asymptotic notation* (Big-Oh classes and relatives).
5. Understand the reasons to make *asymptotic approximations*.
6. Be able to *analyze* simple *algorithms*.

Unit 1: ***Machines & Models***



Outline

What is an algorithm?

An algorithm is a sequence of instructions.

think: recipe

More precisely:

e. g. Python script

1. mechanically executable
   ⇝ no “common sense” needed
2. finite description   ≠ finite computation!
3. solves a *problem*, i. e., a class of problem instances

   $x + y$, not only $17 + 4$

- input-processing-output abstraction

Typical example: *bubblesort*

⇝ not a specific program but the underlying idea

What is a data structure?

A data structure is

1. a rule for encoding data
   (in computer memory), plus
2. algorithms to work with it
   (queries, updates, etc.)

typical example: binary search tree

1.1 Algorithm analysis

Good algorithms

Our goal: Find good (best?) algorithms and data structures for a task.

Good "usually" means

can be complicated in distributed systems

- fast running *time*
- moderate memory *space* usage

Algorithm analysis is a way to

- compare different algorithms,
- predict their performance in an application

Running time experiments

Why not simply run and time it?

- results only apply to
  - single *test* machine
  - tested inputs
  - tested implementation
  - ...
  - ≠ ***universal truths***

- instead: consider and analyze algorithms on an abstract machine
  - ⇝ provable statements for model
  - ⇝ testable model hypotheses

survives Pentium 4

⇝ Need precise model of machine (costs), input data and algorithms.

Data Models

Algorithm analysis typically uses one of the following simple data models:

- **worst-case performance:**
  consider the *worst* of all inputs as our cost metric

- **best-case performance:**
  consider the *best* of all inputs as our cost metric

- **average-case performance:**
  consider the average/expectation of a *random* input as our cost metric

Usually, we apply the above for *inputs of same size n*.

$\rightsquigarrow$ performance is only a **function of** n.

1.2 The RAM Model

Clicker Question

What is the cost of *adding* two d-digit integers?
(For example, for $d = 5$, what is $45\,235 + 91\,342$?)

A constant time

B logarithmic in d

C proportional to d

D quadratic in d

E no idea what you are talking about

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Machine models

The machine model decides

- what algorithms are possible
- how they are described (= programming language)
- what an execution *costs*

Goal: Machine model should be
detailed and powerful enough to reflect actual machines,
abstract enough to unify architectures,
simple enough to analyze.

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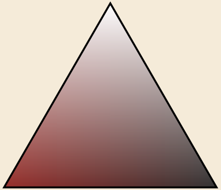
$\rightsquigarrow$ usually some compromise is needed



Random Access Machines

Random access machine (RAM)

more detail in §2.2 of *Sequential and Parallel Algorithms and Data Structures* by Sanders, Mehlhorn, Dietzfelbinger, Dementiev

- unlimited *memory* $\texttt{MEM}[0], \texttt{MEM}[1], \texttt{MEM}[2], \ldots$
- fixed number of *registers* $R_1, \ldots, R_r$ (say $r = 100$)
- memory cells $\texttt{MEM}[i]$ and registers R_i store w-bit integers, i. e., numbers in $[0..2^w - 1]$
  w is the word width/size; typically $\boxed{w \propto \lg n}$ $\rightsquigarrow$ $2^w \approx n$
- Instructions:
  - load & store: $R_i := \texttt{MEM}[R_j]$ $\texttt{MEM}[R_j] := R_i$
  - operations on registers: $R_k := R_i + R_j$ (arithmetic is *modulo* 2^w!)
    also $R_i - R_j$, $R_i \cdot R_j$, $R_i \operatorname{div} R_j$, $R_i \bmod R_j$
    C-style operations (bitwise and/or/xor, left/right shift)
  - conditional and unconditional jumps
- cost: number of executed instructions

we will see further models later

$\rightsquigarrow$ The RAM is the standard model for sequential computation.

Pseudocode

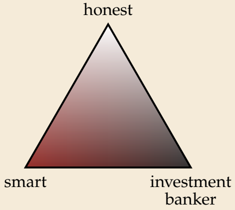
- Programs for the random-access machine are very low level and detailed

$\approx$ assembly/machine language

Typical simplifications when describing and analyzing algorithms:

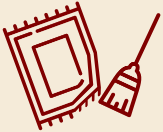
- more abstract *pseudocode* ← code that humans understand (easily)
  - control flow using **if**, **for**, **while**, etc.
  - variable names instead of fixed registers and memory cells
  - memory management (next slide)
- count *dominant operations* (e. g. memory accesses) instead of all RAM instructions

In both cases: We can go to full detail where needed.



Memory management & Pointers

- A random-access machine is a bit like a bare CPU ... without any operating system
  $\rightsquigarrow$ cumbersome to use
- All high-level programming languages add *memory management* to that:
  - Instruction to *allocate* a contiguous piece of memory of a given size (like `malloc`).
    - used to allocate a `new` array (of a fixed size) or
    - a `new` object/record (with a known list of instance variables)
    - There's a similar instruction to `free` allocated memory again.
  - A *pointer* is a memory address (i. e., the i of `MEM`[i]).
  - Support for procedures (a.k.a. functions, methods) calls including recursive calls
    - (this internally requires maintaining call stack)



We will mostly ignore *how* all this works in COMP526.