9 Range-Minimum Queries

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COMP526 (Fall 2022)
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Learning Outcomes

1. Know the *RMQ problem* and its *connection* to longest common extensions in strings.
2. Know and understand trivial RMQ solutions and *sparse tables*.
3. Know and understand the *Cartesian trees* data structure.
4. Know and understand the *exhaustive-tabulation technique* for RMQ with linear-time preprocessing.

Unit 9: *Range-Minimum Queries*

Outline

9 Range-Minimum Queries

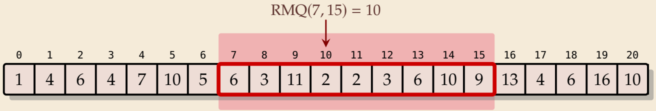
9.1 Introduction
9.2 RMQ, LCP, LCE, LCA — WTF?
9.3 Trivial Solutions & Sparse Tables
9.4 Cartesian Trees
9.5 Exhaustive Tabulation

9.1 Introduction

Range-minimum queries (RMQ)

array / numbers don't change

- **Given:** Static array $A[0..n)$ of numbers
- **Goal:** Find minimum in a range;
  A known in advance and can be preprocessed



- **Nitpicks:**
  - Report *index* of minimum, not its value
  - Report *leftmost* position in case of ties

Clicker Question

Given the array from the slides, what is $\mathrm{RMQ}_A(1, 6)$ = 1

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	4	6	4	7	10	5	6	3	11	2	2	3	6	10	9	13	4	6	16	10

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Rules of the Game

- comparison-based $\rightsquigarrow$ values don't matter, only relative order

- Two main quantities of interest:

  $\rightsquigarrow$ space usage $\le P(n)$

  1. **Preprocessing time**: Running time $P(n)$ of the preprocessing step
  2. **Query time**: Running time $Q(n)$ of one query (using precomputed data)

- Write $\langle P(n), Q(n) \rangle$ **time solution** for short

Clicker Question

What do you think, what running times can we achieve? For a $\langle P(n), Q(n) \rangle$ time solution, enter “<P(n),Q(n)>”.

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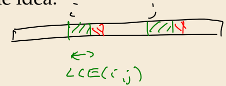
9.2 RMQ, LCP, LCE, LCA — WTF?

Recall Unit 6

Application 4: Longest Common Extensions

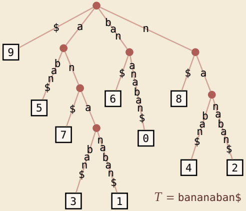
- We implicitly used a special case of a more general, versatile idea:



The ***longest common extension (LCE)*** data structure:

  - **Given:** String $T[0..n-1]$
  - **Goal:** Answer LCE queries, i. e.,
    given positions i, j in T,
    how far can we read the same text from there?
    formally: $\mathrm{LCE}(i,j) = \max\{\ell : T[i..i+\ell) = T[j..j+\ell)\}$

$\rightsquigarrow$ use suffix tree of T!



T = bananaban$

longest common prefix of ith and jth suffix

- In $\mathcal{T}$: $\mathrm{LCE}(i,j) = \mathrm{LCP}(T_i, T_j)$ $\rightsquigarrow$ same thing, different name!
  $=$ string depth of *lowest common ancester (LCA)* of leaves $\boxed{i}$ and $\boxed{j}$

- in short: $\mathrm{LCE}(i,j) = \mathrm{LCP}(T_i, T_j) = \mathrm{stringDepth}(\mathrm{LCA}(\boxed{i}, \boxed{j}))$

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Recall Unit 6

Efficient LCA

How to find lowest common ancestors?

- Could walk up the tree to find LCA $\rightsquigarrow$ $\Theta(n)$ worst case
- Could store all LCAs in big table $\rightsquigarrow$ $\Theta(n^2)$ space and preprocessing

Amazing result: Can compute data structure in $\Theta(n)$ time and space that finds any LCA is **constant(!) time**.

- a bit tricky to understand
- but a theoretical breakthrough
- and useful in practice

and suffix tree construction inside . . .

$\rightsquigarrow$ for now, use $O(1)$ LCA as black box.

$\rightsquigarrow$ After linear preprocessing (time & space), we can find LCEs in $O(1)$ time.

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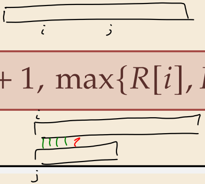
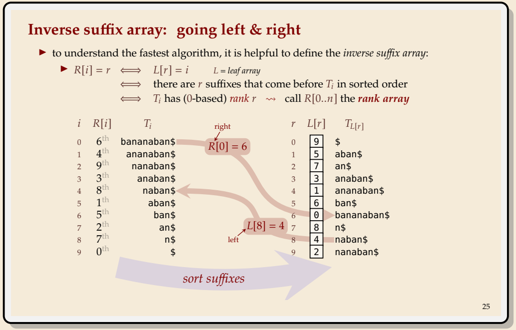
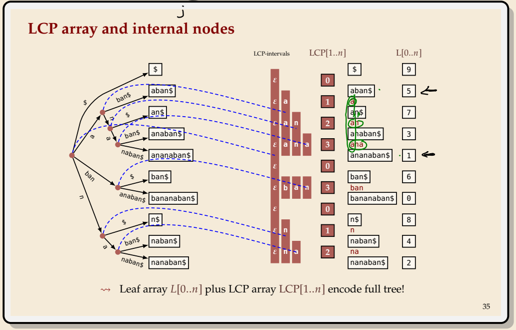
Finally: Longest common extensions

- In Unit 6: Left question open how to compute LCA in suffix trees
- But: Enhanced Suffix Array makes life easier!



$$\mathrm{LCE}(i, j) \;=\; \mathrm{LCP}\Big[\mathrm{RMQ}_{\mathrm{LCP}}\big(\min\{R[i], R[j]\} + 1,\, \max\{R[i], R[j]\}\big)\Big]$$





RMQ Implications for LCE

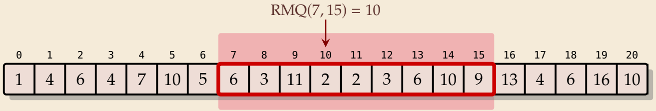
- Recall: Can compute (inverse) suffix array and LCP array in $O(n)$ time

$\rightsquigarrow$ A $\langle P(n), Q(n) \rangle$ time RMQ data structure implies a $\langle P(n) + O(n), Q(n) \rangle$ time solution for longest-common extensions

want: $\langle O(n), O(1) \rangle$

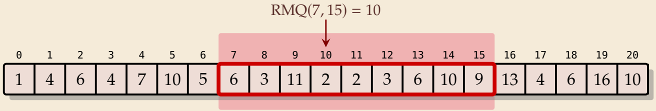
9.3 Trivial Solutions & Sparse Tables

Trivial Solutions



- Two easy solutions show extreme ends of scale:

Trivial Solutions

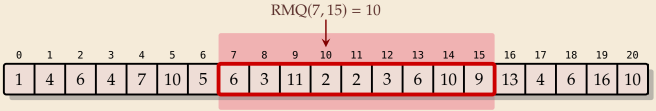


- Two easy solutions show extreme ends of scale:

1. **Scan on demand**
    - no preprocessing at all
    - answer $\mathrm{RMQ}(i, j)$ by scanning through $A[i..j]$, keeping track of min

$\rightsquigarrow \langle O(1), O(n) \rangle$

Trivial Solutions



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2. **Precompute all**
   - Precompute all answers in a big 2D array $M[0..n)[0..n)$
   - queries simple: $\mathrm{RMQ}(i, j) = M[i][j]$

   $\rightsquigarrow \langle O(n^3), O(1) \rangle$

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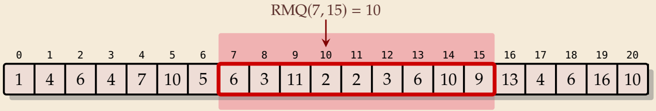

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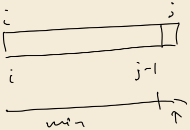
   $\rightsquigarrow$ $\langle O(n^3), O(1)\rangle$
   - Preprocessing can reuse partial results $\rightsquigarrow$ $\langle O(n^2), O(1)\rangle$



Sparse Table

- **Idea:** Like "precompute-all", but keep only some entries
- store $M[i][j]$ iff $\ell = j - i + 1$ is 2^k.
  - $\rightsquigarrow$ $\le n \cdot \lg n$ entries
  - $\rightsquigarrow$ Can be stored as $M'[i][k]$ = $M[i][i+2^k-1]$

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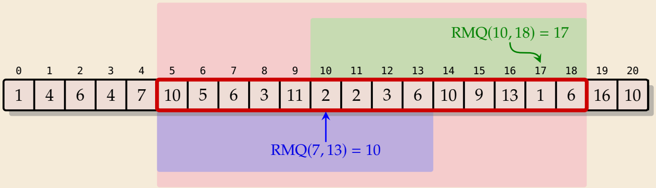
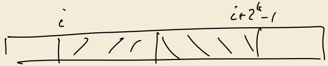


1. Find k with $\ell/2 \leq 2^k \leq \ell$
2. Cover range $[i..j]$ by
   2^k positions right from i and
   2^k positions left from j
3. $\text{RMQ}(i, j) =$
   $\arg\min\{A[rmq_1], A[rmq_2]\}$

   with $rmq_1 = \text{RMQ}(i, i + 2^k - 1) = M'[i][k]$
   $rmq_2 = \text{RMQ}(j - 2^k + 1, j)$
   $= M'[j - 2^k + 1][k]$

Sparse Table



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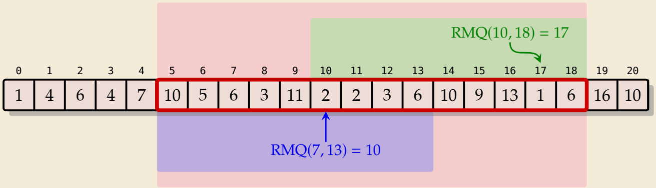
- store $M[i][j]$ iff $\ell = j - i + 1$ is 2^k.
  - ⇝ $\le n \cdot \lg n$ entries
  - ⇝ Can be stored as $M'[i][k]$

$M'[i][k] = \arg\min\{A[M'[i][k-1]], A[M'[i+2^{k-1}][k-1]]\}$

- How to answer queries?



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   with $rmq_1 = \text{RMQ}(i, i + 2^k - 1)$
   $rmq_2 = \text{RMQ}(j - 2^k + 1, j)$

- Preprocessing can be done in $O(n \log n)$ times

⇝ $\langle O(n \log n), O(1) \rangle$ time solution!

9.4 Cartesian Trees

RMQ & LCA

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
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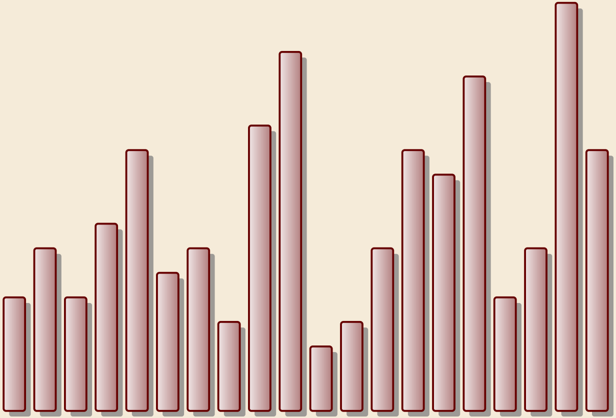
RMQ & LCA



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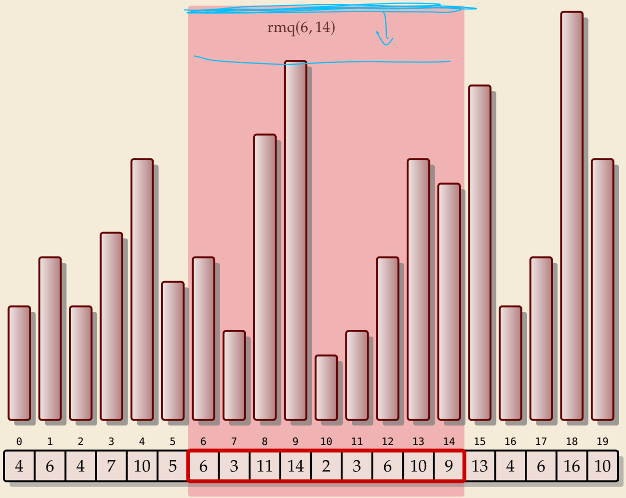
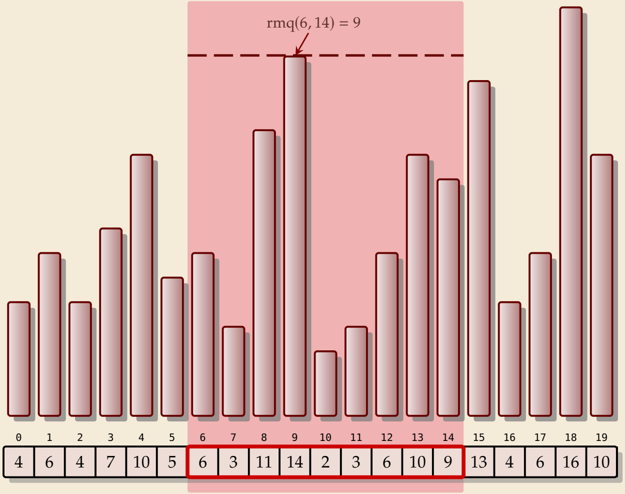


- **Range-max queries** on array A:

$$\text{rmq}_A(i, j) = \underset{i \le k \le j}{\arg\max}\, A[k]$$
$$= \textit{index} \text{ of max}$$

RMQ & LCA



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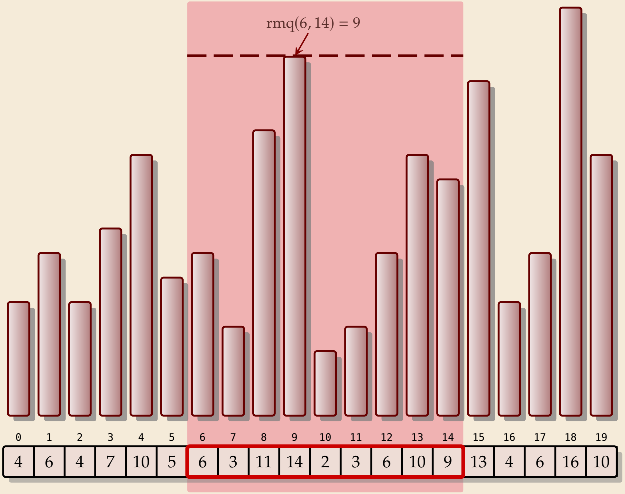

- **<u>R</u>ange-<u>m</u>ax <u>q</u>ueries** on array A:
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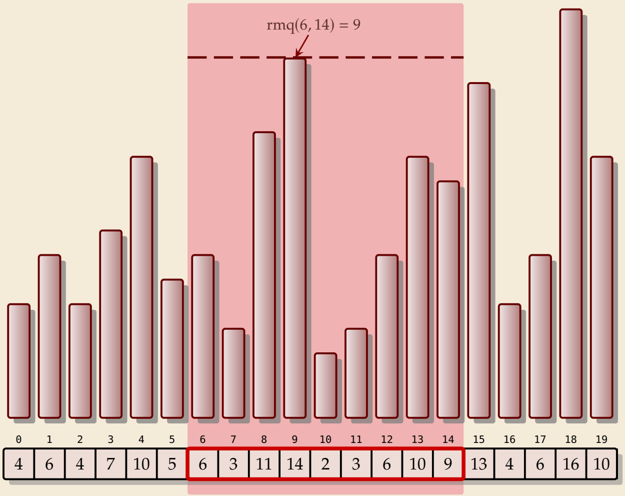
RMQ & LCA



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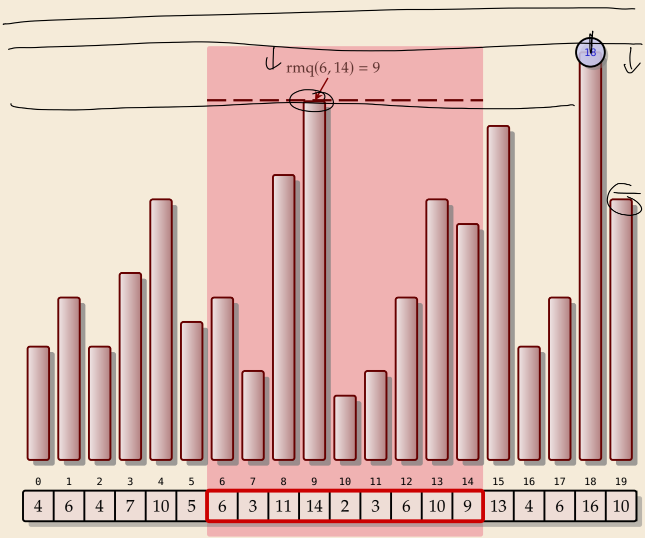
RMQ & LCA



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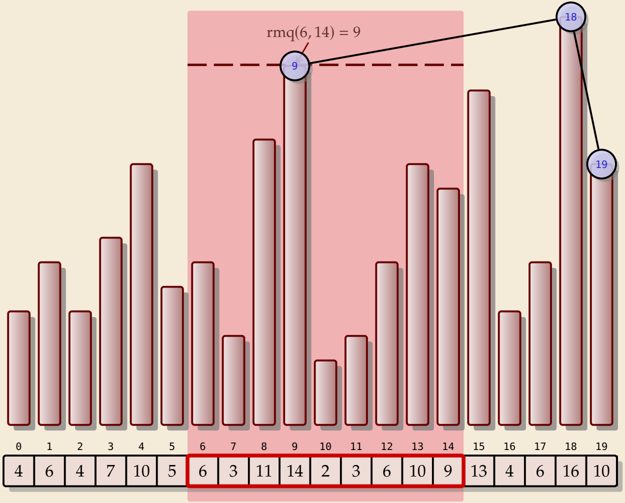

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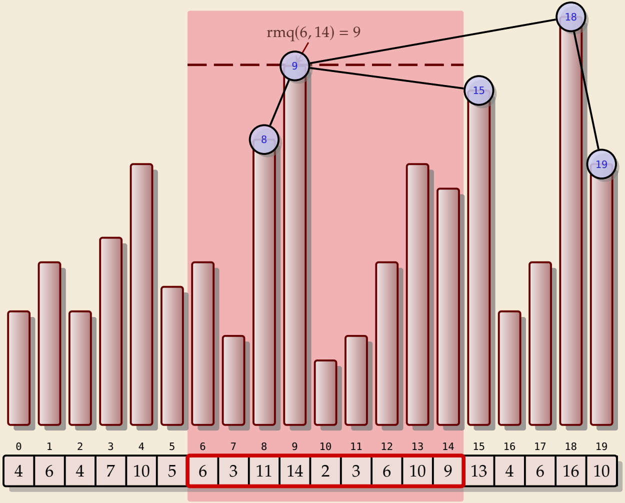

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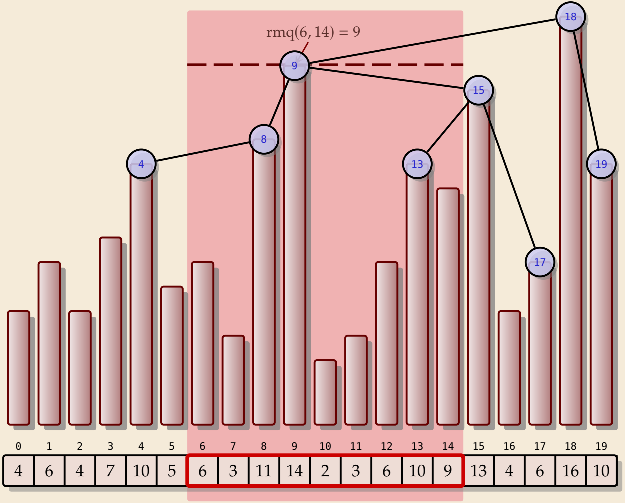

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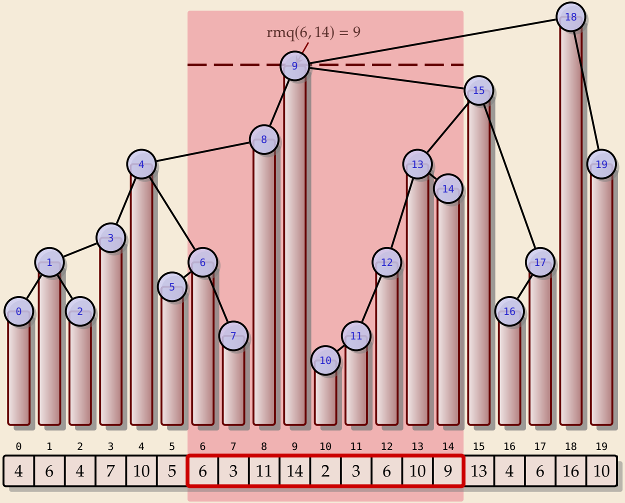

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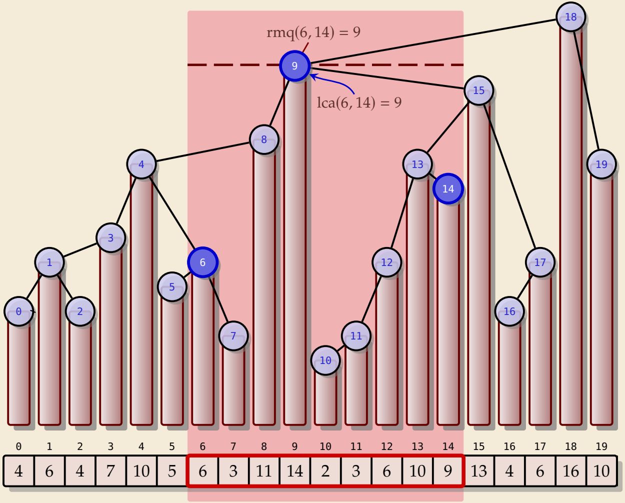

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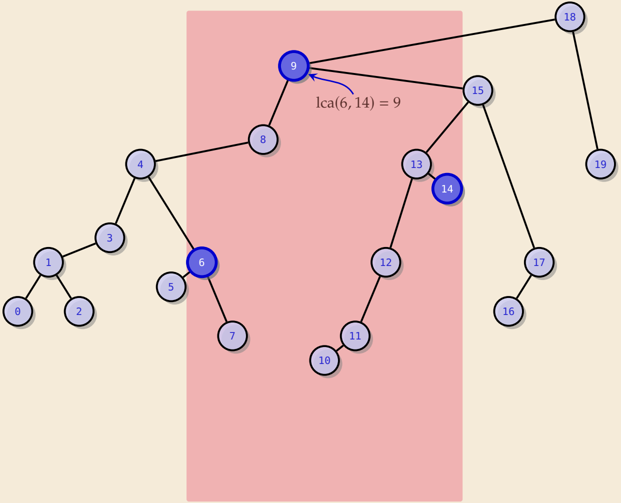
- **Cartesian tree:** (cf. *treap*) construct binary tree by sweeping line down
- $\text{rmq}(i, j)$ = **lowest common ancestor** (LCA)

RMQ & LCA



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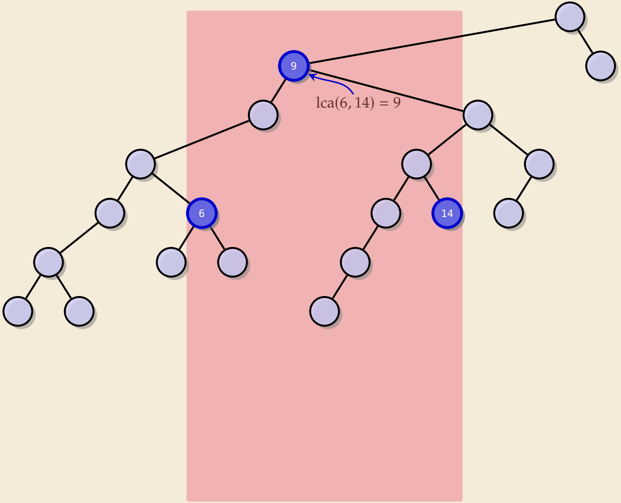

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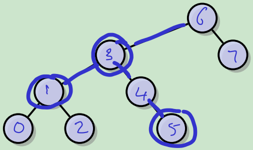
  of ith and jth node in inorder

Clicker Question



Given the (max-oriented) Cartesian tree for A on the left, what is $\mathrm{RMQ}_A(1,5)$?

$= 3$

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Clicker Question

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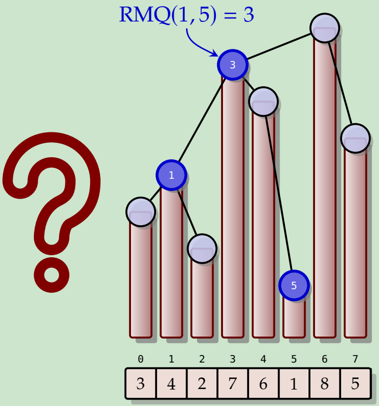

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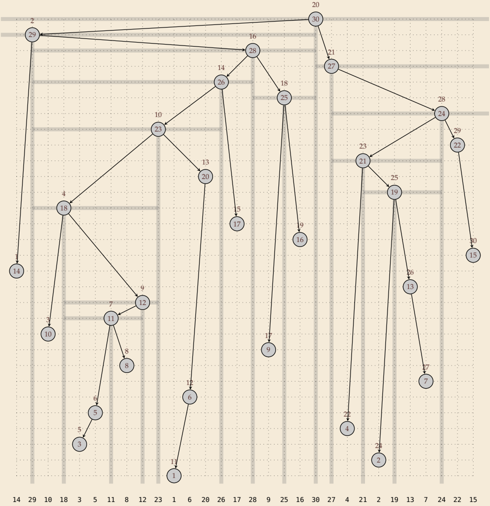
Cartesian Tree – Larger Example



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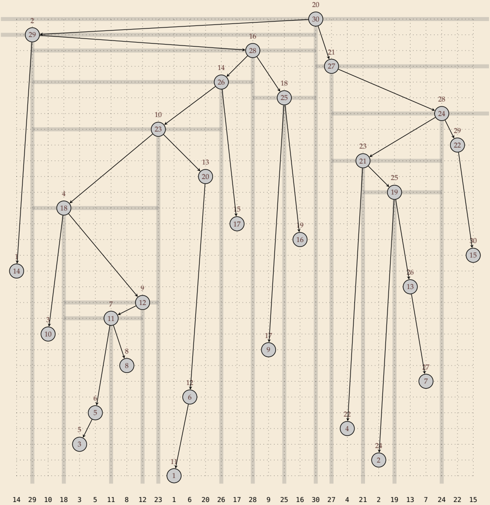

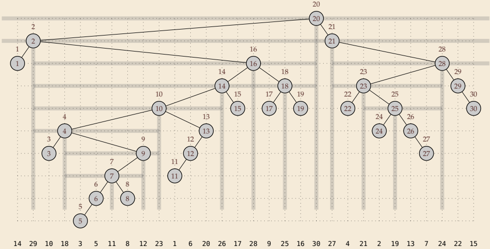

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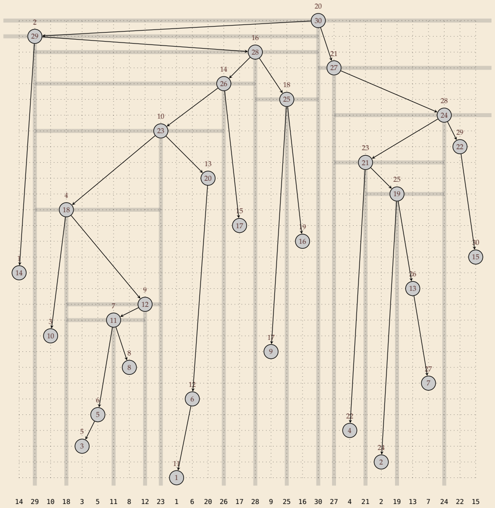
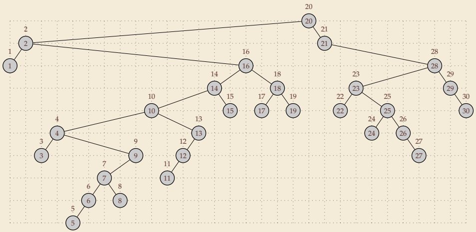
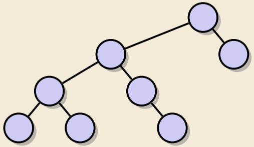




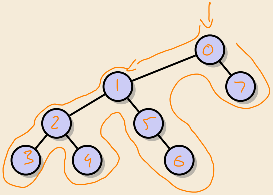
Counting binary trees



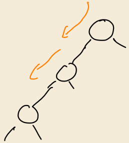
- Given the Cartesian tree, all RMQ answers are determined

  and vice versa!

Counting binary trees



11 11 11 00 00 01 00 00



- Given the Cartesian tree, all RMQ answers are determined

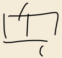
  and vice versa!

- How many different Cartesian trees are there for arrays of length n?
  - known result: *Catalan numbers* $\frac{1}{n+1}\binom{2n}{n}$
  - easy to see: $\leq 2^{2n}$

construct a binary code for binary trees with 2n bits for n-node tree

preorder traversal of tree has left child

store



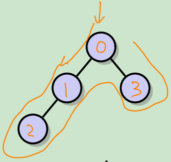
has right child

$\rightsquigarrow$ many arrays will give rise to the same Cartesian tree

Can we exploit that?

Clicker Question

preorder traversal of nodes, store [|]
has left / has right



What binary string corresponds to the tree shown on the left?

(using the encoding just discussed)

11 10 0 0 0 0

→ sli.do/comp526

9.5 Exhaustive Tabulation

Four Russians?

The exhaustive-tabulation technique to follow is often called "Four Russians trick" . . .

- The algorithmic technique was published 1970 by V. L. Arlazarov, E. A. Dinitz, M. A. Kronrod, and I. A. Faradžev
- all worked in Moscow at that time . . . but not even clear if all are Russians!

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- American authors coined the slightly derogatory "Method of Four Russians"
  . . . name in widespread use

Bootstrapping

- We know a $\langle O(n \log n), O(1) \rangle$ time solution
- If we use that for $m = \Theta(n / \log n)$ elements, $O(m \log m) = O(n)$!

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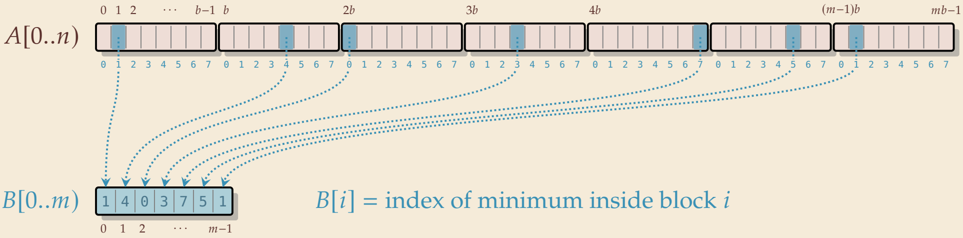
- Create array of block minima $B[0..m)$ for $m = \lceil n/b \rceil = O(n/\log n)$



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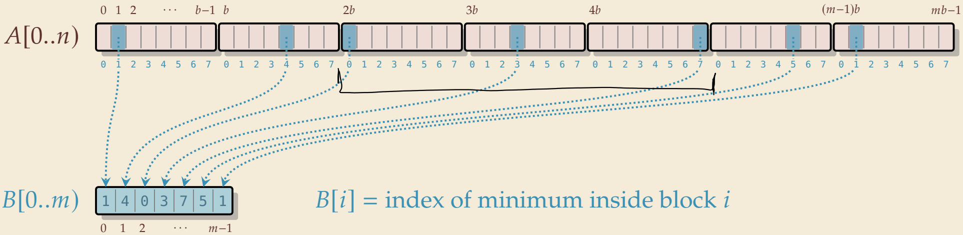
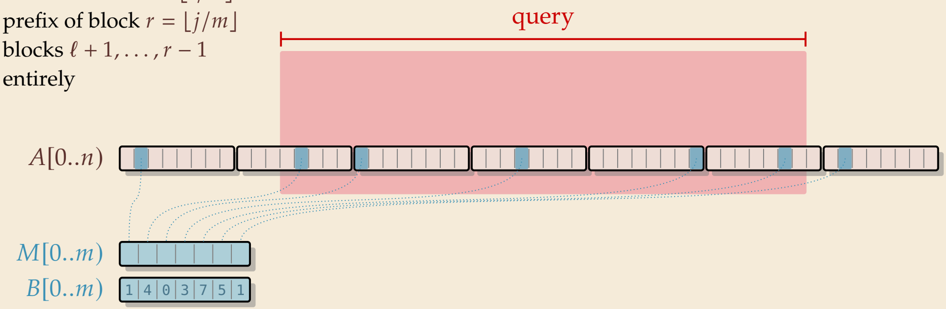


⇝ Use sparse tables for B.

⇝ Can solve RMQs in $B[0..m)$ in $\langle O(n), O(1) \rangle$ time

Query decomposition

- Query $\mathrm{RMQ}_A(i, j)$ covers
  - suffix of block $\ell = \lfloor i/m \rfloor$
  - prefix of block $r = \lfloor j/m \rfloor$
  - blocks $\ell + 1, \ldots, r - 1$ entirely



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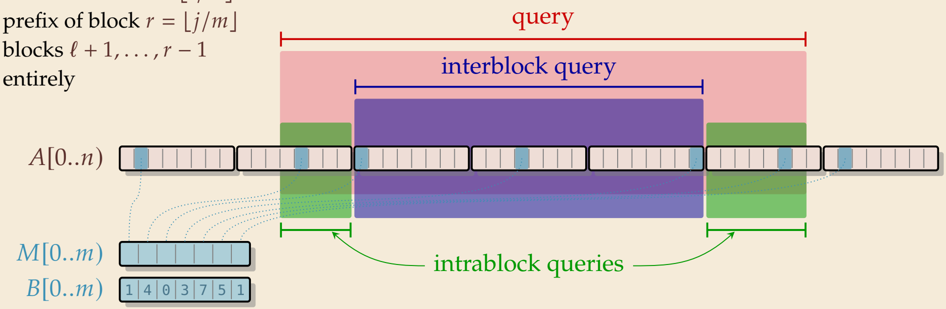

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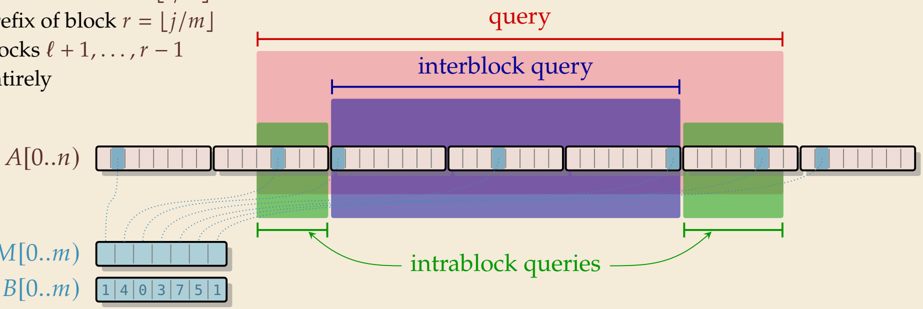

- $\mathrm{RMQ}_A(i,j) = \arg\min_{k\in K} A[k]$ with $K = \left\{ \begin{array}{l} \mathrm{RMQ}_{\text{block } \ell}\big(i-\ell b,\ (\ell+1)b-1\big), \\ b\cdot \mathrm{RMQ}_M\big(\ell+1, r-1\big) + \\ \qquad B\big[\mathrm{RMQ}_M\big(\ell+1, r-1\big)\big], \\ \mathrm{RMQ}_{\text{block } r}\big(rb,\ j-rb\big) \end{array} \right\}$

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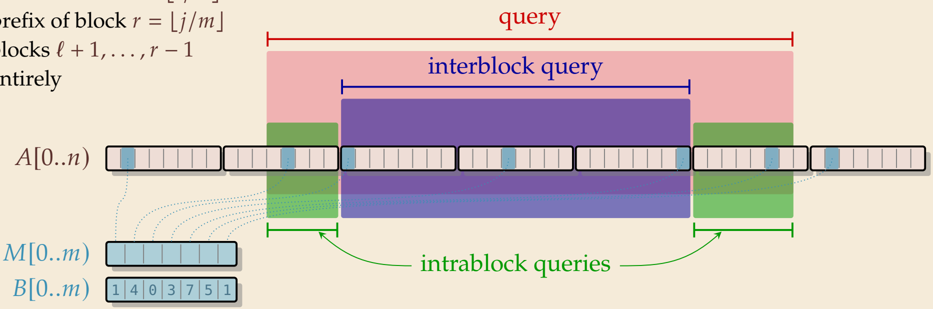

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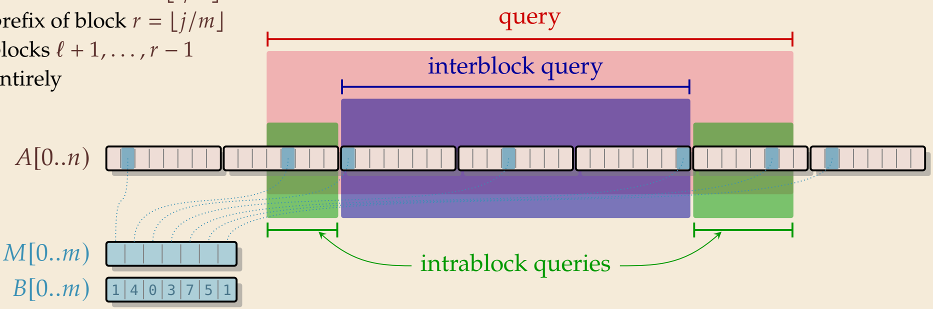

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⇝ only 3 possible values to check
if intrablock and interblock queries known ✓

Intrablock queries [1]

$\rightsquigarrow$ It remains to solve the intrablock queries!

- Want $\langle O(n), O(1) \rangle$ time overall

  ↖ must include preprocessing for all $m = \left\lceil \frac{n}{b} \right\rceil = \Theta\left(\frac{n}{\log n}\right)$ blocks!

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- ⇝ ***Exhaustive Tabulation Technique:***
  1. represent each subproblem by storing its *type* (here: encoding of Cartesian tree)
  2. *enumerate* all possible subproblem types and their solutions
  3. use type as index in a large *lookup table*

Intrablock queries [2]

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   - can be done in linear time

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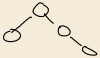
$b = 4$

	Block type	i	j	RMQ(i, j)
	⋮			
11000100		0	2	1
	"	0	3	1
	"	0	1	1
	⋮			

Intrablock queries [2]

1. For each block, compute $2b$ bit representation of Cartesian tree
   - can be done in linear time

2. Compute large lookup table

Block type	i	j	$\mathrm{RMQ}(i, j)$
$\vdots$			
$\vdots$			

- $\le \sqrt{n}$ block types
- $\le b^2$ combinations for i and j

$\rightsquigarrow$ $\Theta(\sqrt{n} \cdot \log^2 n)$ rows

- each row can be computed in $O(\log n)$ time

$\rightsquigarrow$ overall preprocessing: $O(n)$ time!

Discussion

- $\langle O(n), O(1) \rangle$ time solution for RMQ

$\rightsquigarrow$ $\langle O(n), O(1) \rangle$ time solution for LCE in strings!

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Research questions:

- Reduce the space usage
- Avoid access to A at query time