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Date: 2022-09-28
Version: 2022-09-28 14:09

Tutorial 1 for COMP 526 – Efficient Algorithmics, Fall 2022

Problem 1 (Mathematical induction)

Given a sequence of numbers $T(n)$ defined recursively by

$$T(n) \;=\; \begin{cases} 3, & \text{for } n = 0; \\ T(n-1) + 4, & \text{for } n \geq 1. \end{cases} \tag{1}$$

a) Compute the first 6 elements of $T(n)$, i.e., $T(0)$, $T(1)$, $T(2)$, $T(3)$, $T(4)$, and $T(5)$.

b) Make an educated guess about the general pattern that this sequence follows. Write this guess as a *closed form* for $T(n)$, i.e., a formula for $T(n)$ without recursive reference to T.

c) Now formally prove the correctness of your guess using mathematical induction.

Problem 2 (Decreasing potential method)

There are two integral[1] parts of integer division: *the quotient* and *the remainder*. For two integers n, $k > 0$ the quotient (or result) of the integer division "n div k" is defined as the largest integer m with $m \cdot k \leq n$. The remainder of the division is defined as $r = n - m \cdot k$. Note that $0 \leq r < k$. The value r is also known as the result of the *modulo* operation, written "$r = n \bmod k$".

Example: $10 \operatorname{div} 3 = 3$ and $10 \bmod 3 = 1$,
$13 \operatorname{div} 5 = 2$ and $13 \bmod 5 = 3$.

Apply the *decreasing potential method* to prove that the following function $Mod(n, k)$ always terminates when called with parameters $n \in \mathbb{N}$ and $k \in \mathbb{N}$, where $\mathbb{N} = \{1, 2, 3, \ldots\}$.

```
procedure Mod(n, k)
// Input: positive integers n, k.
// Output: value of n mod k.
t := n
while t ≥ k
    t := (t − k)
end while
return t
```

[1] pun intended