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Tutorial 1 for COMP 526 – Applied Algorithmics, Winter 2020

Problem 1 (Orders of magnitude)

Order the following functions with respect to their asymptotic order of magnitude (i.e., their Θ-class).

$$n,\ \sqrt{n},\ n^{1.5},\ n^2,\ n\lg n,\ n\lg\lg n,\ n\lg^2 n,\ n\lg(n^2),\ \frac{2}{n},\ 2^n,\ 2^{n/2},\ 37,\ n^3,\ n^2\lg n.$$

Problem 2 (Loop invariants)

There are two integral[1] parts of integer division: *the quotient* and *the remainder*. For two integers n, $k > 0$ the quotient (or result) of the integer division "$n \operatorname{div} k$" is defined as the largest integer m with $m \cdot k \le n$. The remainder of the division is defined as $r = n - m \cdot k$. Note that $0 \le r < k$. The value r is also known as the result of *modulo* operation, written "$r = n \bmod k$".

Example: $10 \operatorname{div} 3 = 3$ and $10 \bmod 3 = 1$,
$13 \operatorname{div} 5 = 2$ and $13 \bmod 5 = 3$.

Apply the *invariant method* to prove the correctness of the following function $Mod(n, k)$, which is supposed to compute $n \bmod k$, where n and k are two positive integer input parameters of the function.

```
procedure Mod(n, k)
// Input: positive integers n, k.
// Output: value of n mod k.
t := n
while t ≥ k
    t := (t − k)
end while
return t
```

[1] pun intended