

Tutorial 2 for COMP 526 – Applied Algorithmics, Winter 2020

Problem 1 (Decreasing function and amortization method)

Consider again the Mod function from last time:

```
1  procedure Mod( $n, k$ )
2  // Input: positive integers  $n, k$ .
3  // Output: value of  $n \bmod k$ .
4   $t := n$ 
5  while  $t \geq k$ 
6      $t := (t - k)$ 
7  end while
8  return  $t$ 
```

- Apply the *decreasing potential method* to prove that the function $Mod(n, k)$ always terminates.
- Try to establish the time complexity of this procedure.

Problem 2 (Telescoping recurrence and mathematical induction)

Given a complexity function $T(n)$ recursively defined as

$$T(n) = \begin{cases} 3, & \text{for } n = 0; \\ T(n-1) + 4, & \text{for } n \geq 1. \end{cases} \quad (1)$$

Find a *closed form* (without recursive reference) for $T(n)$ by iterating (inserting the recursive definition) until you can make an educated guess.

Then prove the correctness of your guess by mathematical induction.