

## Tutorial 1 for COMP 526 – Applied Algorithmics, Spring 2021

### Problem 1 (Mathematical induction)

Given a sequence of numbers  $T(n)$  defined recursively by

$$T(n) = \begin{cases} 3, & \text{for } n = 0; \\ T(n-1) + 4, & \text{for } n \geq 1. \end{cases} \quad (1)$$

- Compute the first 6 elements of  $T(n)$ , i.e.,  $T(0)$ ,  $T(1)$ ,  $T(2)$ ,  $T(3)$ ,  $T(4)$ , and  $T(5)$ .
- Make an educated guess about the general pattern that this sequence follows. Write this guess as a *closed form* for  $T(n)$ , i.e., a formula for  $T(n)$  without recursive reference to  $T$ .
- Now formally prove the correctness of your guess using mathematical induction.

### Problem 2 (Decreasing potential method)

There are two integral<sup>1</sup> parts of integer division: *the quotient* and *the remainder*. For two integers  $n, k > 0$  the quotient (or result) of the integer division “ $n \operatorname{div} k$ ” is defined as the largest integer  $m$  with  $m \cdot k \leq n$ . The remainder of the division is defined as  $r = n - m \cdot k$ . Note that  $0 \leq r < k$ . The value  $r$  is also known as the result of the *modulo* operation, written “ $r = n \operatorname{mod} k$ ”.

**Example:**  $10 \operatorname{div} 3 = 3$  and  $10 \operatorname{mod} 3 = 1$ ,  
 $13 \operatorname{div} 5 = 2$  and  $13 \operatorname{mod} 5 = 3$ .

Apply the *decreasing potential method* to prove that the following function  $\operatorname{Mod}(n, k)$  always terminates when called with parameters  $n \in \mathbb{N}$  and  $k \in \mathbb{N}$ , where  $\mathbb{N} = \{1, 2, 3, \dots\}$ .

---

```
1  procedure  $\operatorname{Mod}(n, k)$ 
2  // Input: positive integers  $n, k$ .
3  // Output: value of  $n \operatorname{mod} k$ .
4   $t := n$ 
5  while  $t \geq k$ 
6     $t := (t - k)$ 
7  end while
8  return  $t$ 
```

---

<sup>1</sup>pun intended