5 Parallel String Matching

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COMP526 (Spring 2022)
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Learning Outcomes

Unit 5: *Parallel String Matching*

1. Know and apply *parallelization strategies* for embarrassingly parallel problems.
2. Identify *limits of parallel speedups*.
3. Understand *string matching by duels*, both sequential and parallel (excluding preprocessing).

Outline

Parallelizing string matching

- We have seen a plethora of string matching methods
- But all efficient methods seem inherently sequential
  Indeed, they became efficient only after building on knowledge from previous steps!

Sounds like the *opposite* of parallel!

⇝ This unit:
  - How well can we parallelize string matching?
  - What new ideas can help?

Here: string matching = find *all* occurrences of P in T (more natural problem for parallel)
always assume $m \leq n$

5.1 Elementary Tricks

Embarrassingly Parallel

- A problem is called *"embarrassingly parallel"*
  if it can immediately be split into *many, small subtasks*
  that can be solved completely *independently* of each other
- Typical example: sum of two large matrices (all entries independent)

⇝ best case for parallel computation (simply assign each processor one subtask)

- Sorting is not embarrassingly parallel
  - no obvious way to define many *small* (=efficiently solvable) subproblems
  - but: some subtasks of our algorithms are, e. g., comparing all elements with pivot

Clicker Question

Is the string-matching problem “embarrassingly parallel”?

- **A** Yes
- **B** No
- **C** Only for $n \gg m$
- **D** Only for $n \approx m$

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Elementary parallel string matching

Subproblems in string matching:

- string matching = check all guesses $i = 0, \ldots, n - m - 1$
- checking one guess is a subtask!

Elementary parallel string matching

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Approach 1:

- Check all guesses in parallel

$\rightsquigarrow$ **Time**: $\Theta(m)$ using sequential checks
$\Theta(\log m)$ on CREW-PRAM ($\rightsquigarrow$ see tutorials)
$\Theta(1)$ on CRCW-PRAM ($\rightsquigarrow$ see tutorials)

} parallel AND

$\rightsquigarrow$ **Work**: $\Theta((n - m)m)$ $\rightsquigarrow$ not great . . .

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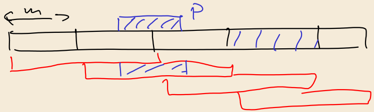

Approach 2:

- Divide T into **overlapping** blocks of $2m$ characters:
  $T[0..2m), T[m..3m), T[2m..4m), T[3m..5m) \ldots$
- Find matches inside blocks in parallel, using efficient sequential method
  $\rightsquigarrow$ $\Theta(2m + m) = \Theta(m)$ each

$\rightsquigarrow$ **Time**: $\Theta(m)$ **Work**: $\Theta(\frac{n}{m} \cdot m) = \Theta(n)$

$\Theta(n+m) \underset{m \le n}{=} \Theta(n)$

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Elementary parallel matching – Discussion

- 👍 very simple methods
- 👍 could even run distributed with access to part of T
- 👎 parallel speedup only for $m \ll n$

Goal:

- work-efficient methods with better parallel time? $\rightsquigarrow$ higher speedup

$\rightsquigarrow$ must genuinely parallelize the matching process! (and the preprocessing of the pattern)

$\rightsquigarrow$ need new ideas

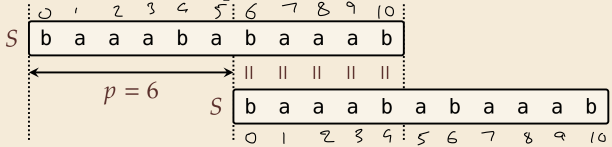
5.2 Periodicity

Periodicity of Strings

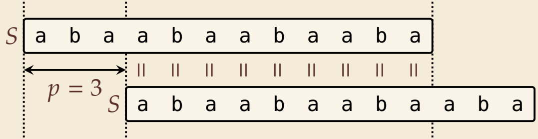
- $S = S[0..n-1]$ has *period* p iff $\forall i \in [0..n-p) : S[i] = S[i+p]$
- $p = 0$ and any $p \geq n$ are trivial periods   but these are not very interesting ...

Examples:

- $S =$ baaababaaab has period 6:



- $S =$ abaabaabaaba has period 3:



Periodicity and KMP

Lemma 5.1 (Periodicity = Longest Overlap)

$p \in [1..n]$ is the *shortest* period in $S = S[0..n-1]$
iff $S[0..n-p)$ is the longest prefix that is also a suffix of $S[p..n)$. ◀

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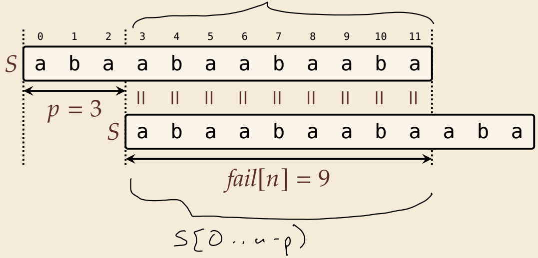
$S[p..n)$

$S[0..n-1]$ has minimal period $p \iff fail[n] = n - p$



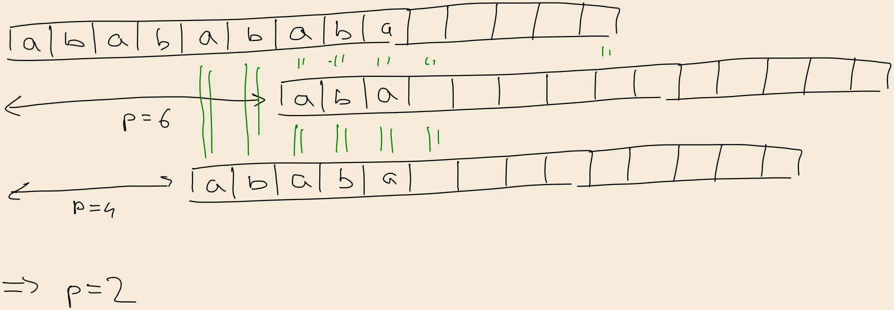
Periodicity Lemma

Lemma 5.2 (Periodicity Lemma)

If string $S = S[0..n-1]$ has periods p and q with $p + q \le n$,
then it has also period $\gcd(p, q)$.

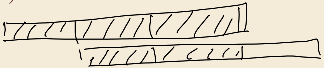
greatest common divisor

Proof:



Periodic strings

- What does the smallest period p tell us about a string $S[0..n)$?
- Two distinct regimes:



1. S is *periodic*: $p \le \frac{n}{2}$
   More precisely: S is totally determined by a string $F = F[0..p) = S[0..p)$
   S keeps repeating F until n characters are filled

   $\rightsquigarrow$ S is highly repetitive!

2. S is *aperiodic* (also *non-periodic*): $p > \frac{n}{2}$
   S **cannot** be written as $S = F^k \cdot Y$ with $k \ge 2$ and Y a prefix of F

Clicker Question

Is S = `aaaaaaaaaab` a periodic string?

- **A** Yes
- **B** No

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Clicker Question

aa…ab
aaa…b

Is S = aaaaaaaaaaab a periodic string? $p = n$

A ~~Yes~~

B No ✓

⇝ “looking repetitive” is not enough for periodic!

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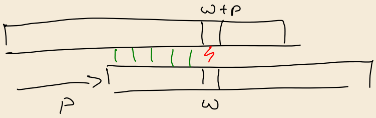
5.3 String Matching by Duels

Periods and Matching



Witnesses for non-periodicity:

- Assume, $P[0..m)$ does **not** have period p

$\rightsquigarrow$ $\exists$ *witness against periodicity*: position $\omega \in [0..m-p)$: $P[\omega] \neq P[\omega+p]$

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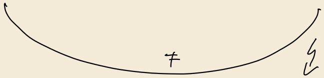
Dueling via witnesses:

- If $P[0..m)$ does **not** have period p, then
  at most one of positions i and $i + p$ can be (the first position of) an occurrence of P.

  Proof: Cannot have $T[(i+p)+\omega] = P[\omega] \neq P[\omega + p] = T[i + (\omega + p)]$.



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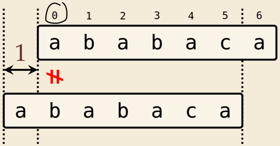
- **Duel** between guess i and $i+p$: compare text character overlapped with witness ω

Dueling example

1. Compute witnesses against periodicity for $P =$ `ababaca`

P aperiodic



p	1
$\omega[p]$	0

Dueling example

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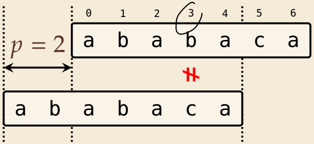

p	1	2
$\omega[p]$	0	3

Dueling example

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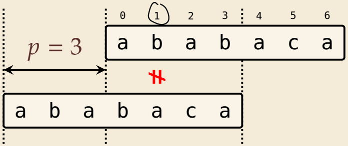

p	1	2	3
$\omega[p]$	0	3	1

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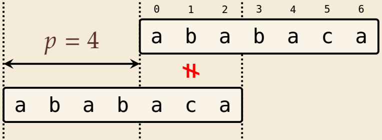

p	1	2	3	4
$\omega[p]$	0	3	1	1

Dueling example

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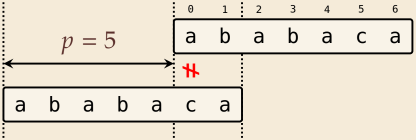
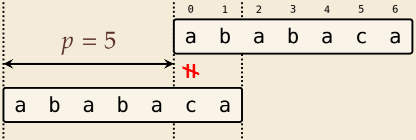


p	1	2	3	4	5
$\omega[p]$	0	3	1	1	0

Dueling example

1. Compute witnesses against periodicity for P = ababaca

p=6 smallest period



p	1	2	3	4	5
$\omega[p]$	0	3	1	1	0

2. Duel! T = abababaaaca

Dueling example

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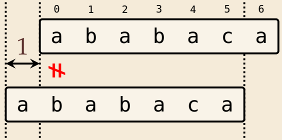

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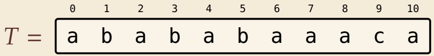

2. Duel! T = abababaaaca

- **0 vs. 1**

  $p = 1, \omega = 0 \rightsquigarrow T[1] = \mathtt{b} \neq P[\omega] \rightsquigarrow$ No occurrence at 1!

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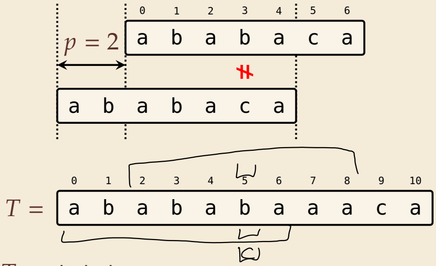

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     $p = 1, \omega = 0 \rightsquigarrow T[1] = \texttt{b} \neq P[\omega] \rightsquigarrow$ No occurrence at 1!
   - **0 vs. 2**
     $p = 2, \omega = 3 \rightsquigarrow T[5] = \texttt{b} \neq \texttt{c} = P[\omega + p] \rightsquigarrow$ No occurrence at 0!

Dueling example

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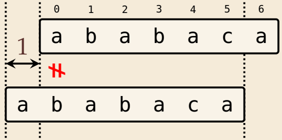

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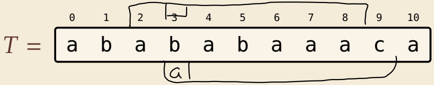

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     $p = 1, \omega = 0 \rightsquigarrow T[1] = \texttt{b} \neq P[\omega] \rightsquigarrow$ No occurrence at 1!
   - **0 vs. 2**
     $p = 2, \omega = 3 \rightsquigarrow T[5] = \texttt{b} \neq \texttt{c} = P[\omega + p] \rightsquigarrow$ No occurrence at 0!
   - **2 vs. 3**
     $p = 1, \omega = 0 \rightsquigarrow T[3] = \texttt{b} \neq \texttt{a} = P[\omega] \rightsquigarrow$ No occurrence at 3!

String Matching by Duels – Sequential

Assume that pattern P is *aperiodic*.

(can deal with periodic case separately; details omitted)

=> these exist

Algorithm:

1. Set $\mu := \lfloor \frac{m}{2} \rfloor$
2. Compute witnesses $\omega[1..\mu]$ against periodicity for all $p \le \frac{m}{2}$.
3. For each block of μ consecutive indices $[0..\mu)$, $[\mu..2\mu)$, $[2\mu..3\mu)$, ...
   run $\mu - 1$ duels to eliminate all but one guess in the block
4. check remaining $\lceil \frac{n}{\mu} \rceil = O(n/m)$ guesses naively

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Analysis:

1. $O(1)$
2. $O(m) \rightsquigarrow$ later
3. $O(\frac{n}{m})$ blocks
   $O(m)$ duels each
4. $O(\frac{n}{m})$,
   $\le m$ cmps each

$\rightsquigarrow$ another worst-case $O(n + m)$ string matching method!

String Matching by Duels – Parallel

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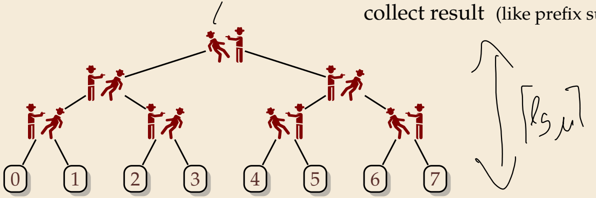
How to parallelize:

1. —
2. $O(\log^2(m)) \rightsquigarrow$ later
3. blocks in parallel (indep.),
   tournament of $\lceil \lg \mu \rceil$ rounds
4. check in parallel
   collect result (like prefix sum)

Tournament of duals:

- each dual eliminates one guess
- $\rightsquigarrow$ declare other guess *winner*
- conceptually like (prefix) sum!



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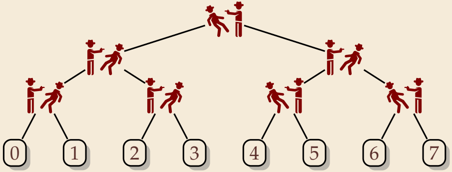

$\rightsquigarrow$ Matching part can be done in $O(\log m)$ parallel time and $O(n)$ work!

Computing witnesses

It remains to find the witnesses $\omega[1..\mu]$.

sequentially:

- an elementary procedure is similar in spirit to KMP failure array
- can be computed in $\Theta(m)$ time

parallel:

- much more complicated $\rightsquigarrow$ beyond scope of the module
  - first $O(\log^2(m))$ time on CREW-RAM
  - later $O(\log m)$ time and $O(m)$ work using *pseudoperiod method*

Parallel Matching – State of the art

- $O(\log m)$ time & work-efficient parallel string matching
  - this is optimal for CREW-PRAM
- on CRCW-PRAM: matching part even in $O(1)$ time ($\rightsquigarrow$ tutorials)
  but preprocessing requires $\Theta(\log \log m)$ time

for your curiosity