

Tutorial 1 for COMP 526 – Applied Algorithmics, Spring 2022

Problem 1 (Mathematical induction)

Given a sequence of numbers $T(n)$ defined recursively by

$$T(n) = \begin{cases} 3, & \text{for } n = 0; \\ T(n-1) + 4, & \text{for } n \geq 1. \end{cases} \quad (1)$$

- Compute the first 6 elements of $T(n)$, i.e., $T(0)$, $T(1)$, $T(2)$, $T(3)$, $T(4)$, and $T(5)$.
- Make an educated guess about the general pattern that this sequence follows. Write this guess as a *closed form* for $T(n)$, i.e., a formula for $T(n)$ without recursive reference to T .
- Now formally prove the correctness of your guess using mathematical induction.

Problem 2 (Decreasing potential method)

There are two integral¹ parts of integer division: *the quotient* and *the remainder*. For two integers $n, k > 0$ the quotient (or result) of the integer division “ $n \operatorname{div} k$ ” is defined as the largest integer m with $m \cdot k \leq n$. The remainder of the division is defined as $r = n - m \cdot k$. Note that $0 \leq r < k$. The value r is also known as the result of the *modulo* operation, written “ $r = n \operatorname{mod} k$ ”.

Example: $10 \operatorname{div} 3 = 3$ and $10 \operatorname{mod} 3 = 1$,
 $13 \operatorname{div} 5 = 2$ and $13 \operatorname{mod} 5 = 3$.

Apply the *decreasing potential method* to prove that the following function $\operatorname{Mod}(n, k)$ always terminates when called with parameters $n \in \mathbb{N}$ and $k \in \mathbb{N}$, where $\mathbb{N} = \{1, 2, 3, \dots\}$.

```
1  procedure Mod( $n, k$ )
2  // Input: positive integers  $n, k$ .
3  // Output: value of  $n \operatorname{mod} k$ .
4   $t := n$ 
5  while  $t \geq k$ 
6     $t := (t - k)$ 
7  end while
8  return  $t$ 
```

¹pun intended